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| RESEARCH ARTICLE

Numerical Analysis of the Effects of Flow Parameter on Magneto-convective Flow Past an Inclined Conductor

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| ABSTRACT
The study of the effects of flow parameters on magneto-convective flow past an inclined conductor with a constant transversal magnetic field applied is considered. The governing momentum and energy equations are solved using a finite central scheme approximation, and the developing partial difference equations are solved using MATLAB software. The study indicated that heat absorption coefficient, Hartmann and Rayleigh numbers greatly influence the flow, and the angle of inclination of the conductor strongly affects the buoyancy force component. The results are represented in 2-dimensional graphs and tables.

Nomenclature

ω	Angle of inclination.	**Pr**	Prandtl number
B_0	Magnetic field strength	**T**	Thermodynamic temperature.
Ha	Hartmann Number	**N**	Buoyancy ratio

| KEYWORDS

Magneto-convective flow, Magnetic field, Richardson number

| ARTICLE INFORMATION

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1. Introduction

Magneto hydrodynamic flow, which is the movement of conductive fluids within a magnetic field, is of great importance both internationally and locally in the marine and aircraft departments, among other sectors of engineering, where it's gained immense application. In the department of Marine and Aircraft, many researchers have attempted to find out the problem of developing ships that are not mechanically driven by other forms of energy, such as fuels, among others. The MHD convective flow knowledge can be taken advantage of, and the knowledge can be used to create ships that are very fast, easy to operate, economical, and environmentally friendly. Consequently, how to build up the induced force during MHD flow has been the foundation of most of the research work.

Nasrin (2011) investigated the influence of a centred conductor on MHD mixed convection in a wavy chamber. The findings indicated that the Richardson number (Ri) and the diameter (D) of the conducting obstacle greatly affect the flow phenomenon. Arash Karimipour *et al.* [2020] investigated the magnetic field influence on combined convection heat transfer in a lid-driven rectangular enclosure. The results of the study showed that the Hartmann number significantly affects the fluid flow structure and temperature field. Sigey *et al.* (2013) studied MHD free convective flow past an infinite vertical porous plate with Joule heating effect. The result showed that an increase in the Joule heating parameter causes an increase in temperature and velocity profile. Sarris *et al.* (2005) investigated MHD free convection in a sideways and volumetrically heated square cavity. The results indicated that the flow profile is influenced by the Rayleigh and Hartmann numbers. Mahdy *et al*. (2013) investigated MHD combined convection in a tilted lid–driven enclosure with opposing thermal buoyancy force. It was concluded that the flow is reduced by the presence of a magnetic field, and the average Nusselt number is an increasing function of the magnetic field angle. Oztop *et al*. (2016) investigated the combined convection of a nano–fluid–filled cavity with oscillating lid under the influence of an inclined magnetic field. The results showed that Strouhal, Richardson, and Hartmann numbers greatly affect the heat transfer process. Moreau *et al*. (1992) investigated natural convection in a rectangular enclosure with a horizontal magnetic field. The results showed that the velocity profile varies with the Hartmann number. From the above literature, it can be seen that there is no significant information on the effect of the angle of inclination of a centred conductor on MHD free and forced convection.

The specific objectives of the study are:

i. To develop a mathematical model for the effects of an inclined conductor on MHD natural and forced convection in a heated wavy chamber.
ii. To evaluate the influence of the angle of inclination on flow distribution.
iii. To determine the influence of the magnetic field on the temperature distribution profile.
iv. To determine the effects of the flow parameter on temperature and velocity profile distribution.

1.1 Model Specification.

The set-up below consists of a two-dimensional lid-driven rectangular chamber of length L. The upper and bottom surfaces of the cavity are insulated, the left lid is at a uniform velocity, and the temperature T_i (50˚C), while the right wall has a temperature T_h(100˚C). The conductor is tilted through an angle (ω) for (30˚≤ ω≤90˚) along the horizontal axis within a conductive fluid taken as air of (Pr = 0.71). A constant magnetic field of strength B_o is applied in the horizontal direction to the bodysides of the chamber.

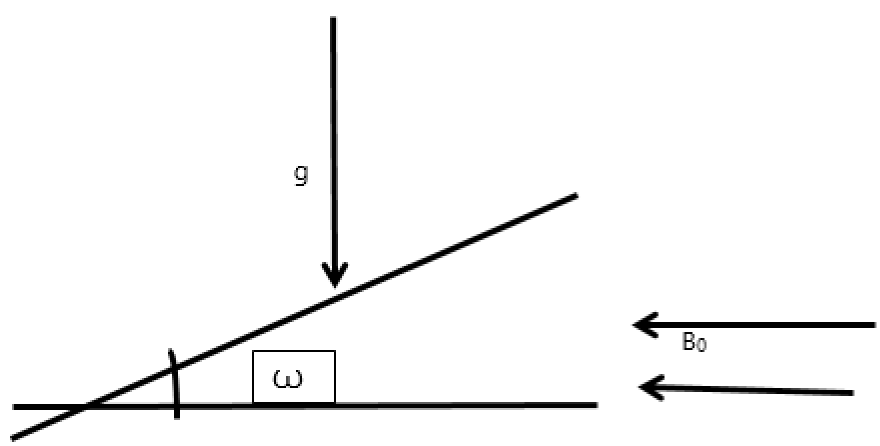


Figure 1: Schematic diagram

B_0 is the external magnetic field strength.

Mathematical formulation

The flow is assumed to be laminar, steady, and incompressible, and the magnetic effect only acts horizontally.

Natural Convection

The specific momentum equation for this case will be obtained by considering Newton's second law. The external forces acting on the fluid in this case are the buoyancy and magnetic force.

Equation along x–axis

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\Pr\left[\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right]+\sin\omega\left(Ra_T\Pr\left[\theta-\mathrm{N}C\right]\right) \quad (1)$$

Equation along y-axis

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\Pr\left[\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right]+\cos\omega\left[Ra_T\Pr\left(\theta-\mathrm{N}C\right)-Ha^2\Pr V\right] \quad (2)$$

EnergyEquation.

The specific energy equation is obtained from the first law of Thermodynamics where Φ is the heat absorption coefficient.

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left[\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right]+\Phi\theta \quad (3)$$

1.2 Forced Convection

In this type of convection, there is a mass movement or fluid movement due to an external forcing condition. The dimensionless equation for this case becomes:

Equation along x -axis

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\Pr\left[\frac{\partial^2 U}{\partial X^2}\right]+\sin\omega \text{ ....} \quad (4)$$

Equation along y -axis

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=\Pr\left[\frac{\partial^2 V}{\partial Y^2}\right]+\cos\omega-Ha^2\Pr V \text{ ....} \quad (5)$$

Method of solution

The issue under study generates partial differential equations whose findings are determined by using the finite difference approach. In this approach, the derivatives developed in the generated differential equations are to be worked out using the Finite Difference (central scheme) method. MATLAB simulation software is used to develop results.

Discretization of the Momentum Equation along x axis

We consider the Momentum Equation along the x-axis (2) and use it to investigate the fluid velocity profiles. For the central scheme (CDS), the values u_x u_y u_{xx} and u_{yy} are replaced by the central difference approximation.

$$u\left(\frac{U_{i+1,j}-U_{i-1,j}}{2\Delta x}\right)+v\left(\frac{U_{i,j+1}-U_{i,j-1}}{2\Delta y}\right)=\left(\frac{U_{i+1,j}-2U_{i,j}+U_{i-1,j}}{(\Delta x)^2}+\frac{U_{i+1,j+1}-2U_{i,j+1}+U_{i-1,j+1}}{(\Delta y)^2}\right)+\sin\omega\left(Ra_T\Pr\left(\theta-N_c\right)\right)$$

(6)

Take $u = v = N_C = \theta = 1$, multiply both sides by $2(\Delta x)$ and let Pr = 0.71 and take $\Delta x = \Delta y = 0.1$ on a square mesh into Eq (2), we get the scheme;

$$0.86U_{i+1,j} + 8U_{i,j} - 1.14U_{i-1,j} = -0.86U_{i,j+1} + 1.14U_{i,j-1} + \sin\omega\left(Ra_T \Pr\left(\theta - N_c\right)\right) \quad (7)$$

Taking i = 1,2,3.......5 and j = 1, we form the following systems of linear algebraic equations,

And with initial and boundary conditions $U_{i,0} = U_{0,j} = 1$ and $U_{i,2} = 0$ respectively, the above algebraic equations (7) can be written in matrix form as;

$$\begin{bmatrix} 8 & 0.86 & 0 & 0 & 0 \\ -1.4 & 8 & 0.86 & 0 & 0 \\ 0 & -1.4 & 8 & 0.86 & 0 \\ 0 & 0 & -1.4 & 8 & 0.86 \\ 0 & 0 & 0 & -1.4 & 8 \end{bmatrix} \begin{bmatrix} U_{1,1} \\ U_{2,1} \\ U_{3,1} \\ U_{4,1} \\ U_{5,1} \end{bmatrix} = \begin{bmatrix} 0.14 + 0.7\sin\omega Ra_T \\ 0.07 + 0.7\sin\omega Ra_T \\ 0.07 + 0.7\sin\omega Ra_T \\ 0.07 + 0.7\sin\omega Ra_T \\ 0.07 + 0.7\sin\omega Ra_T \end{bmatrix} \quad (8)$$

We use equation (8) to investigate the effects of ω and Ra_T on the vertical fluid velocity profile.

Discretization of the Momentum Equation along the y-axis

We consider the Momentum Equation along a long y-axis (2) and use it to investigate the fluid velocity profiles. For the central scheme (CDS), the values, V_x, V_y, V_{xx} and V_{yy} are replaced by the central difference approximation. On substituting these values into Equation (2), it gives;

$$u\left(\frac{V_{i+1,j} - V_{i-1,j}}{2\Delta x}\right) + v\left(\frac{V_{i,j+1} - V_{i,j-1}}{2\Delta y}\right) = \left(\frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta x)^2} + \frac{V_{i+1,j+1} - 2V_{i,j+1} + V_{i-1,j+1}}{(\Delta y)^2}\right) + \cos\omega\left(Ra_T \Pr\left(\theta - N_c\right)\right) - 0.7Ha^2 V_{i,j}$$

(9)

Take $u = v = N_C = \theta = 1$, multiply both sides by $2(\Delta x)$, and let Pr = 0.71 and take $\Delta x = \Delta y = 0.1$ on a square mesh into Eq (9), we get the scheme;

$$0.86V_{i+1,j} + \left(8 + Ha^2\right)V_{i,j} - 1.14V_{i-1,j} = -0.86V_{i,j+1} + 1.14V_{i,j-1} + \cos\omega\left(Ra_T \Pr\left(\theta - N_c\right)\right) \quad (10)$$

Taking and i = 1,2,3.......5 and j = 1 we form the following systems of linear algebraic equations and with initial and boundary conditions $V_{i,0} = V_{0,j} = 1$ and $V_{i,2} = 0$ respectively, the above equation in its matrix form comes to:

$$\begin{bmatrix} (8+Ha^2) & 0.86 & 0 & 0 & 0 \\ -1.4 & (8+Ha^2) & 0.86 & 0 & 0 \\ 0 & -1.4 & (8+Ha^2) & 0.86 & 0 \\ 0 & 0 & -1.4 & (8+Ha^2) & 0.86 \\ 0 & 0 & 0 & -1.4 & (8+Ha^2) \end{bmatrix} \begin{bmatrix} V_{1,1} \\ V_{2,1} \\ V_{3,1} \\ V_{4,1} \\ V_{5,1} \end{bmatrix} = \begin{bmatrix} 0.14 + 0.7\cos\omega Ra_T \\ 0.07 + 0.7\cos\omega Ra_T \\ 0.07 + 0.7\cos\omega Ra_T \\ 0.07 + 0.7\cos\omega Ra_T \\ 0.07 + 0.7\cos\omega Ra_T \end{bmatrix} \quad (11)$$

We use equation (11) to investigate the effects of Ha^2, ω and Ra_T on the horizontal fluid velocity profile.

Discretization of Energy Equation

We consider Energy Equation (3) and use it to investigate the fluid temperature distribution. For the central scheme (CDS), the values, $\theta_x, \theta_x, \theta_{xx}$ and θ_{yy} are replaced by central difference approximation. When these values are substituted into Equation (3), we get;

$$u\left(\frac{\theta_{i+1,j}-\theta_{i-1,j}}{2\Delta x}\right)+v\left(\frac{\theta_{i,j+1}-\theta_{i,j-1}}{2\Delta y}\right)=\left(\frac{\theta_{i+1,j}-2\theta_{i,j}+\theta_{i-1,j}}{(\Delta x)^2}+\frac{\theta_{i+1,j+1}-2\theta_{i,j+1}+\theta_{i-1,j+1}}{(\Delta y)^2}\right)+\Phi\theta_{i,j} \quad (12)$$

Take $u = v = N_C = \theta = 1$, multiply both sides by $2(\Delta x)$ and let Pr = 0.71 and take $\Delta x = \Delta y = 0.1$ on a square mesh into Eq (12), we get the scheme;

$$-19\theta_{i+1,j}+(80-0.2\phi)\theta_{i,j}-21\theta_{i-1,j}=19\theta_{i,j+1}+21\theta_{i,j-1} \quad (13)$$

Taking and i = 1,2,3.......5 and j = 1, we form the following systems of linear algebraic equations, with initial boundary conditions and with initial and boundary conditions $\Theta_{i,0}=\Theta_{0,j}=1$ and $\Theta_{i,2} = 0$ respectively, the above algebraic equation in its matrix form becomes:

$$\begin{bmatrix} (8-0.2\Phi) & 0.86 & 0 & 0 & 0 \\ -1.4 & (8-0.2\Phi) & 0.86 & 0 & 0 \\ 0 & -1.4 & (8-0.2\Phi) & 0.86 & 0 \\ 0 & 0 & -1.4 & (8-0.2\Phi) & 0.86 \\ 0 & 0 & 0 & -1.4 & (8-0.2\Phi) \end{bmatrix}\begin{bmatrix}\theta_{1,1}\\ \theta_{2,1}\\ \theta_{3,1}\\ \theta_{4,1}\\ \theta_{5,1}\end{bmatrix}=\begin{bmatrix}420\\210\\210\\210\\210\end{bmatrix} \quad (14)$$

We use equation (14) to investigate the effects of Φ, on the temperature distribution.
The next chapter details the results obtained for the effects of ω, Φ and Ra_T on the horizontal, vertical fluid velocity profiles and temperature distribution. The results are presented in tables and discussed graphically.

3. Results and Discussion

3.1 Effect of Rayleigh number on horizontal fluid velocity profile

We hold constant the values of $\omega = 45^0$ and solve equation (11) for values of varying values of

Ra_T = 1000, 2000 and 3000 in equation (11), we obtain the solutions of Ra_T as presented in Table 1.

Table 1 shows horizontal fluid velocity with varying Rayleigh number.

Rayleigh number	Chamber length, x				
	0	1	2	3	4
$Ra_T = 1000$	54.969867	64.39719864	66.10490750	65.56742555	73.35829483
$Ra_T = 2000$	109.9240769	128.786 853	132.200311	131.1230422	146.7075683
$Ra_T = 3000$	164.8767 57	193.1704766	198.293755	196.6797998	220.0527557

The results in Table 1 are represented graphically in Figure 1.

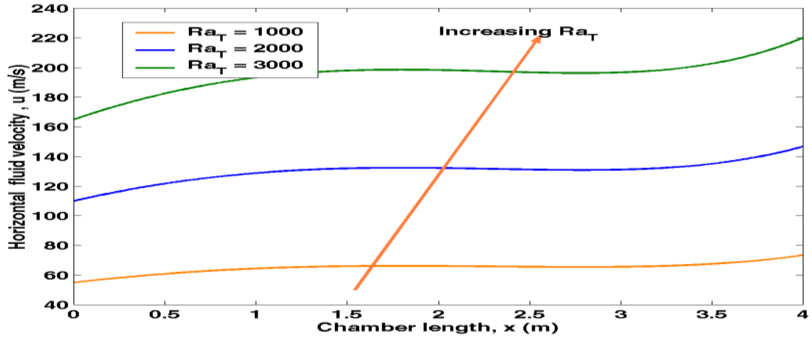


Figure 1 Horizontal velocity against chamber length with varying Rayleigh number

The influence of the Rayleigh number on horizontal fluid velocity can be observed from Figure 1. A raise in the Rayleigh number leads to an increase in the horizontal fluid velocity. The result also indicates a conducting dominating regime at low Rayleigh number with vertical velocity profiles and a convective dominating regime at high Rayleigh number with horizontal velocity profiles.

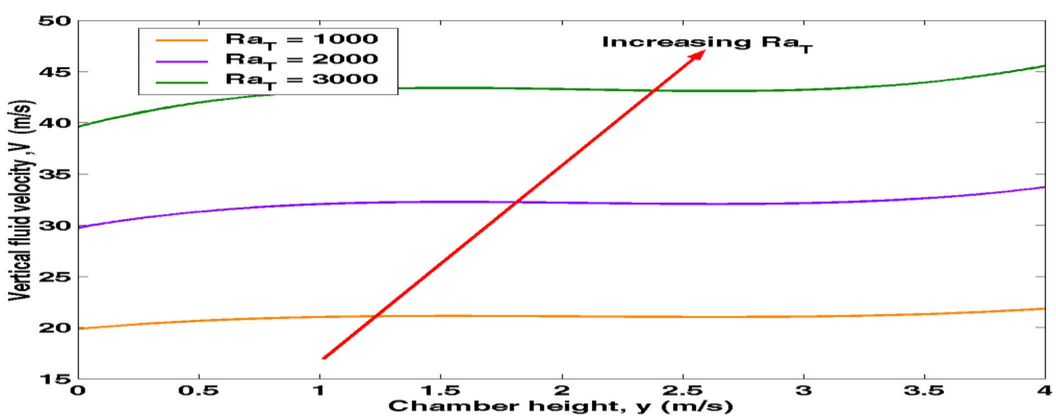
3.2 Effect of Rayleigh number on vertical fluid velocity profile: We hold constant the values of $\omega = 45^0$ and solve equation (8) for values of varying values of Ra_T = 1000, 2000 and 3000 in equation (8)**,** we obtain the solutions of Ra_T as presented in the table 2

From Figure 2 below, it can be concluded that the energy of the vertical velocity profiles increases as the Rayleigh number (*Ra*) increases. The result also shows a conducting dominating regime at low Rayleigh number with vertical velocity profiles and a convective dominating regime at high Rayleigh number.

Table 2. Effect of Rayleigh number on vertical fluid velocity profile.

Rayleigh number	**Chamber height, y(m)**				
	0	**1**	**2**	**3**	**4**
$Ra_T = 1000$	19.87721432	21.031356494	21.099543722	21.077565	21.8573449
$Ra_T = 2000$	29.74487769	32.0595853	32.19609 311	32.1480422	33.711568
$Ra_T = 3000$	39.62095 57	43.08773766	43.29263757	43.220557995	45.5657758

The results in Table 2 are represented graphically in Figure 2



3.3 Effects of Hartmann number on vertical velocity profile

We hold constant the values of Ra_T = 1000, $\omega = 45^0$ and solve equation (8) for values of varying values of Ha = 4, 5 and 6 in equation (8), for varying values of Ha, we obtain solutions as presented in Table 3 below.

Table 3. Vertical fluid velocity with varying Hartmann number

Hartman number	Chamber height, y(m)				
	0	**1**	**2**	**3**	**4**
$Ha = 4$	0.02242373	0.02008426	0.01942971	0.018014275	0.01652252
$Ha = 5$	0.02183564	0.01907728	0.01840992	0.017035289	0.01552758
$Ha = 6$	0.0212922	0.01806648	0.017411235	0.01605638	0.01456488

The results in Table 3 are represented graphically as in Figure 3 below;

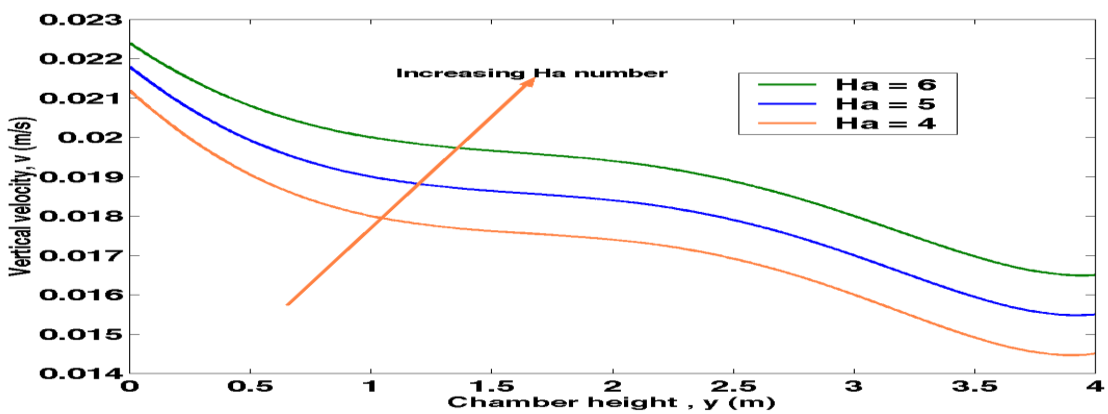


Figure 3. Vertical velocity against chamber length with varying the Hartman number

From the above, it can be deduced that the velocity is affected by Hartmann number. This is because of the formation of a thinner boundary zone, which shows a general trend to induce an electromotive force in the free stream flow of MHD, hence the velocity rises as shown above.

3.4 Effect of dimensionless absorption coefficient on temperature distribution

We varying Φ = 10, 15, and 20 in equation (14). Solving equation (14) for varying values of Φ , we obtain solutions for varying Φ as presented in Table 4 below.

Table 4. Fluid temperature distribution for varying dimensionless absorption coefficient

Dimensionless absorption coefficient	Chamber length, x				
	0	**1**	**2**	**3**	**4**
$\Phi = 10$	6.813856	5.872194	5.5395637	5.1702643	4.0835628
$\Phi = 15$	6.941472	6.021216	5.6722278	5.3011632	4.172579
$\Phi = 20$	7.069552	6.170012	5.82 3442	5.4426483	4.263325

The results in Table 4 are represented graphically as shown in Figure 4 below;

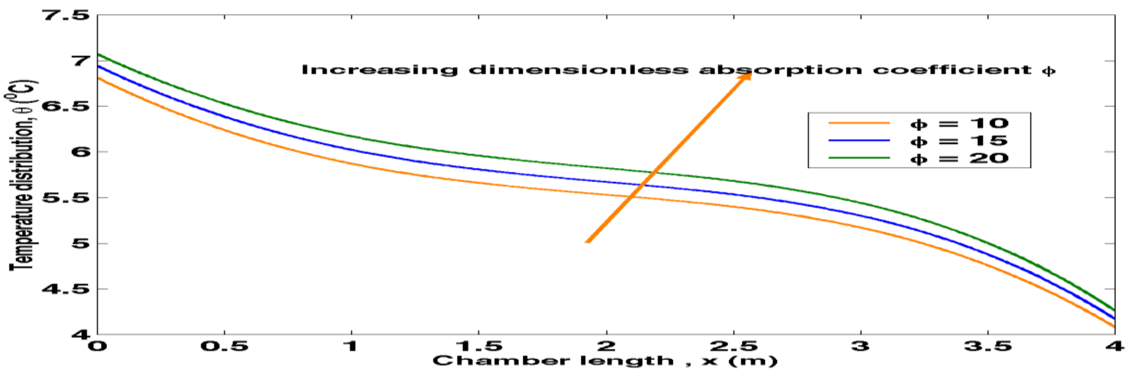


Figure 4 Temperature against chamber length with varying dimensionless absorption coefficient: shows that for a given value of dimensionless absorption coefficient, the surface heat flow tends to reduce with an increase in Φ. Besides, slightly away from the point of temperature dispersion, the temperature dispersion is considerable, and eventually, as we move far away from the plate, the outcome is found to be reducing.

4. Conclusion

A numerical study was conducted to analyze the influence of the effects: Rayleigh number, Hartmann number, and absorption coefficient on fluid velocity profiles and temperature distribution. The following results can be concluded from the study;

- The flow is greatly dependent on both Hartmann and Rayleigh numbers.
- Heat absorption coefficient greatly affects the temperature profile, thereby affecting the velocity profile distribution.
- The Hartmann numbers strongly affect the induced magnetic force. This, in turn, affects the fluid flow velocity.

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