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| RESEARCH ARTICLE

Development of a Mathematical Model for Palm Fruit Digester Design: Integrating Dimensional Analysis and Process Optimization

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| ABSTRACT

This investigation delineates a thorough methodology for the formulation of a parametric model about a palm fruit digester apparatus, employing the tenets of dimensional analysis. The palm fruit digester, an essential component within the palm oil manufacturing processes, encompasses intricate interactions among various parameters that influence its operational efficiency and throughput. A palm fruit digester was conceptualised and fabricated locally to attain a more profound comprehension of these interrelationships. The performance parameter was scrutinized as data were amassed during the apparatus's testing, and the Buckingham Pi theorem was utilized to derive dimensionless Pi terms from pertinent parameters. The results of our analysis indicated that R^2 values of 0.9956 for both throughput and machine efficiency were achieved. Consequently, the model can forecast the efficiency and throughput of the palm fruit digester. This helps researchers understand that the palm fruit digester's operational efficiency and throughput are essential to the industry's sustainability, financial stability, and quality standards. In addition to helping to optimise these procedures, the dimensional analysis modelling approach advances strategic ideas that have the potential to reshape operational capabilities in the processing of palm oil.

| KEYWORDS

Buckingham Pi theorem, Digester, Dimensional analysis, Palm oil production, Parametric modelling.

| ARTICLE INFORMATION

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1. Introduction

Palm oil has become an indispensable commodity, extensively used worldwide in the food, cosmetic, and biofuel industries. This ubiquity stems from its unique properties, including high oxidative stability, an ideal balance of saturated and unsaturated fatty acids, and versatility in various applications. As global demand for palm oil escalates, so does the pressure on the industry to adopt sustainable practices and ensure efficient extraction processes. In 2022, the global palm oil market reached a valuation of USD 67.3 billion, with projections showing a compound annual growth rate (CAGR) of 5.1% from 2023 to 2030, indicating a promising potential for industry growth. In Nigeria, palm oil production was estimated at 1.4 million metric tons in 2022 alone. Over the period from 2009 to 2022, the production quantity consistently showed an upward trend, with significant growth observed in 2010, registering an increase of approximately 14 percent [Sasu, 2024; Erhimona, 2023].

The heart of palm oil extraction lies in the palm fruit digester, a vessel designed to break down the fruit's cellular structure and release the oil-rich contents. The efficiency of the digester significantly impacts the overall oil extraction process, making its design and operation crucial to the industry's success and environmental impact.

Traditional approaches to designing palm fruit digesters often rely on empirical methods or trial-and-error procedures, which may lead to suboptimal designs. With the growing computational capabilities and the advent of parametric modelling techniques, researchers now have powerful tools at their disposal to analyze complex systems and explore a vast design space. Parametric modelling is a broad term used to describe various modelling approaches that involve defining and manipulating parameters to create, modify, or analyze models [Janssen, 2015]. Parametric modelling is a computational technique that involves defining a set of parameters that govern a system's behaviour and performance. These parameters can be adjusted independently, allowing engineers to explore different design configurations quickly and accurately. By leveraging parametric modelling, engineers can simulate the behaviour of the system under various conditions, optimize its performance, and discover previously unexplored design possibilities.

In the context of a palm fruit digester, parametric modelling proves to be valuable for simulating and studying various factors that influence the oil extraction process. Variables such as temperature, pressure, moisture content, fruit size, digester geometry, etc., can be analyzed using this approach. By understanding the interactions of these variables, engineers can identify the optimal combination that maximizes oil yield, minimizes energy consumption, and ensures an environmentally sustainable process. Thus, dimensional analysis serves as a powerful mathematical tool to simplify complex relationships between variables. It enables engineers to identify the significant variables that affect a system's behaviour and group them into dimensionless groups. The foundation of this process lies in Buckingham's Pi theorem, which states that if a physical relationship involves 'n' variables and 'm' fundamental dimensions, then there are 'n-m' dimensionless groups that fully characterize the system's behaviour. Thus, this study aims to investigate and develop an optimized parametric model for a palm fruit digester using dimensional analysis. Through this approach, the goal is to enhance oil extraction efficiency, improve throughput, and promote sustainable practices within the palm oil industry. By understanding the key factors that impact the digester's performance, engineers can design and operate the system in a manner that maximizes efficiency while minimizing environmental impact.

2. Literature Review

Palm oil, an essential commodity, is widely used across the globe in the food, beauty, and biofuel sectors. It finds application in various products such as margarine, cosmetics, candles, lipstick bases, waxes, polishes, sweets, medicines, plating, lubricants, biodiesel, spreads, ice cream, coffee additives, creams, fatty acid formulation, palm-based cheese, encapsulated goods, milk substitutes, condiments, and oil extraction [Erhimona, 2023]. Despite its high demand, local industries still rely heavily on conventional methods for designing palm fruit digesters, often based on trial-and-error or empirical techniques, which can result in less effective designs. To enhance design accuracy, industries should consider adopting a parametric model approach [Ukwu, 2024; Onyenanu, 2024]. Numerous researchers have explored the use of parametric models to improve the design performance of agricultural machinery.

Ikejiofor [2016] studied a mathematical model for predicting the throughput capacity of a cocoyam chipper. The study employed the Buckingham pi theorem to create this model. The outcomes of their investigation demonstrated a strong alignment between the model they constructed and the actual experimental data, showcasing a high R-square value of 0.91. Asonye [2019] researched a mathematical model for predicting the cutting energy of cocoyam (colocasia esculenta). This investigation utilized the Buckingham pi theorem to construct the model. The model they formulated underwent validation against real experimental data, revealing a substantial coefficient of determination, denoted as R^2 = 0.982, indicating a strong relationship between the projected and actual values. The resulting predictive model was proven effective in calculating the necessary cutting energy for cocoyam cormels, reaching up to 98% accuracy, and studied Mathematical modelling of the performance of an impact snake gourd (Trichosanthes cucumerina. L) seed sheller. Their findings revealed a significant correlation between the predicted and experimental values, with a high coefficient of determination percentages observed: 98.45% for the impact of moisture content on decortication efficiency, 99.69% for the impact of hammer diameter on decortication efficiency, and 97.35% for the impact of hammer speed on decortication efficiency. This strong alignment between predictions and experimental outcomes underscores the suitability of the model they developed. [Ikejiofor, 2022] researched a mathematical model for predicting the power requirement of a cocoyam chipper. Hence, the objective of this research is to examine and create an optimized parametric model for a palm fruit digester by employing the Buckingham pi theorem.

3. Predictive Model Development

Dimensional analysis, a well-developed methodology in physical sciences and engineering studies, was employed to develop the predictive equations for the efficiency and throughput models of the palm fruit digester machine. This evidence suggests that any physically significant equation must exhibit dimensional homogeneity, ensuring the validity of the relationship between actual physical quantities regardless of the magnitudes of the base unit.

3.1 Efficiency model

The efficiency of the palm fruit digester machine "η" depends on the density of the palm fruit, Volume of the digester drum, Blade angle, Number of blades, Machine torque, and blade thickness. Therefore, the drying rate "η" in terms of dimensionless parameters is expressed as:

Table 1 Variables Affecting the Efficiency

S/N	Variables	Symbol	Unit	Dimension
1	Efficiency	η	%	$M^0L^0T^0$
2	Density of the palm fruit	ρ	Kgm^{-3}	ML^{-3}
3	Volume of the digester drum	V_D	m^3	L^3
4	Blade angle	θ	(o)	M L^0T^0
5	Number of blades	N	-	M L^0T^0
6	Machine torque	T_m	N-m	L^2T^{-2}
7	Blade thickness	t_B	mm	L

η (ρ,V_D, θ, NT_m, t_B) equ.1

Note that ***dependent variables = independent variables***

However, the functional relationship between the dependent and independent variables can be written as: f(ρ,V_D, θ, N, T_m, t_B) = 0 .... equ. 2

The total number of variables, n, is equal to 7.

Number of fundamental dimensions = 3 (MLT)

Therefore, the number of π-term = n-m = 4

However, there will be four $\pi_1, \pi_2, \pi_{3,} \pi_4$

$\pi_1 = C_e f(\pi_2, \pi_{3,} \pi_4)$ .... equ. 3

Where; C_e=Efficiency constant, $\pi_1\pi_2\pi_3\pi_4$= Pi terms to be determined

Table 2: Dimensional matrix of variables for efficiency model

Dimensional Unit	Variables						
	η	ρ	V_D	θ	N	T_m	t_B
	a	b	c	d	e	f	g
M	0	1	0	0	0	1	0
L	0	0	3	0	0	2	1
T	0	-3	0	0	0	-2	0

The π-terms can be determined by considering the corresponding dimensional expression in the equation 4, 5, 6, 7

$[M^0L^0T^0]^a = [ML^{-3}]^b[L^3]^c[M^0L^0T^0]^d[M^0L^0T^0]^e[ML^2T^{-2}]^f[L]^g$ .... equ. 4

M = b + f =0 .... equ. 5

L = 3c + 2f + g = 0 .... equ. 6

T = -3b -2f = 0 .... equ. 7

Assume arbitrary values of a =1, b = 1, c = 0, d = 0, to obtain π_1:

M = (0) + (0) = 0 .... equ. 8

L= 3(0) + 2(0) + (0) .... equ. 9

T = -3(0) – 2(0) = 0 .... equ. 10

$[M^0L^0T^0]^1 = [ML^{-3}]^0[L^3]^0[M^0L^0T^0]^0[M^0L^0T^0]^0[ML^2T^{-2}]^0[L]^0$ .... equ. 11

π_1 = ŋ .... equ. 12

To obtain π_2, assume an arbitrary value of a = 0, b = 0, c = 1, f = 0

M = (0) + (0) = 0 .... equ. 13

L= 3(1) + 2(0) + g = 0 .... equ. 14

T = -3(0) – 2(0) = 0 .... equ. 15

g = -3

$[M^0L^0T^0]^0 = [ML^{-3}]^0[L^3]^1[M^0L^0T^0]^0[M^0L^0T^0]^0[ML^2T^{-2}]^0[L]^{-3}$ .... equ. 16

$\pi_2 = \frac{V}{{t_B}^2}$ .... equ. 17

To obtain π_3, assume an arbitrary value of a = 0, b = 0, e = 1, g = 0

M = (0) + (0) = 0 .... equ. 18

L= 3(0) + 2(0) + (0) .... equ. 19

T = -3(0) – 2(0) = 0 .... equ. 20

$[M^0L^0T^0]^1 = [ML^{-3}]^0[L^3]^0[M^0L^0T^0]^0[M^0L^0T^0]^1[ML^2T^{-2}]^0[L]^0$ .... equ. 21

π_3 = N .... equ. 22

To obtain π_4, assume an arbitrary value of a = 0, b = 0, c = 1, d = 1

M = (0) + (0) = 0 .... equ. 23

L= 3(0) + 2(0) + (0) .... equ. 24

T = -3(0) – 2(0) = 0 .... equ. 25

$[M^0L^0T^0]^1 = [ML^{-3}]^0[L^3]^0[M^0L^0T^0]^1[M^0L^0T^0]^0[ML^2T^{-2}]^0[L]^0$ .... equ. 26

$\pi_4 = \theta$ .... equ. 27

According to [Shafii, 1996], combining the dimension terms to reduce it to a manageable level can be achieved by multiplication and division. However, we have the equation 27 as:

$\boldsymbol{\pi_5 = \frac{\pi_4 \ x \ \pi_3}{\pi_2}}$ .... equ. 28

$\pi_5 = \frac{t_B{}^2 N\theta}{V_B}$ .... equ. 29

In order to determine the Ƞ-model, equate equation 29 into equation 3. However, the efficiency model can be expressed as:

Ƞ $= C_e \left(\frac{t_B{}^2 N\theta}{V_B}\right)$ .... equ. 30

3.2 Throughput model

The throughput of the palm fruit digester machine "T_p" depends on the Mass of boiled palm fruit, Digestion time, Machine power, Drum diameter, Size of the palm fruit pieces, and the Viscosity of the extracted palm oil. Therefore, the drying rate."T_p" in terms of dimensionless parameters is expressed as:

Table 3: Variables affecting the Throughput of the machine

S/N	Variables	Symbol	Unit	Dimension
1	Throughput	T_p	Kg/sec	MT^{-1}
2	Mass of boiled palm fruit	M	Kg	M
3	Digestion time	T_t	Sec	T
4	Machine Power	P	Watt	ML^2T^{-3}
5	Drum diameter	D	mm	L
6	Size of the palm fruit pieces	S	mm	L
7	Viscosity of the extracted palm oil	μ	Pa.sec	$ML^{-1}T^{-1}$

$T_p = f(M, T_t, P, D, S, \mu)$ .... equ .31

Note that ***dependent variables = independent variables***

However, the functional relationship between the dependent and independent variables can be written as:

$f\ (T_p, M, T_t, P, D, S, \mu) = 0$ .... equ .32

The total number of variables, n, is equal 7.

Number of fundamental dimensions = 3 (MLT)

Therefore, the number of π-term = n-m = 4

However, there will be four $\pi_{11}, \pi_{22}, \pi_{33,}\pi_{44}$

$\pi_{11} = C_t f(\pi_{22}, \pi_{33,}\pi_{44})$ .... equ .33

Where; C_t = Throughput constant, $\pi_{11}\pi_{22}\pi_{33}\pi_{44}$ = Pi terms to be determined

Table 4: Dimensional matrix of variables for throughput model

Dimensional Unit	Variables						
	T_p	M	T_t	P	D	S	μ
	a	b	c	d	e	f	g
M	1	1	0	1	0	0	0
L	0	0	0	2	1	1	-1
T	-1	0	1	-3	0	0	-1

$[MT^{-1}]^a = [\mathrm{M}]^b[\mathrm{T}]^c[ML^2T^{-3}]^d[\mathrm{L}]^0[\mathrm{L}]^e[ML^{-1}T^{-1}]^f$ .... equ .34

M = a + b + g .... equ .34

L = 2d + e + f – g .... equ .35

T = -a + c -3d – g .... equ .36

Assume arbitrary values of a =1, b = 0, c = 0, d = 0, to obtain π_{11}:

M = (1) + (b) + (0) = 0 .... equ .37

L= 2(0) + e + f – g .... equ .38

T = -(0) + c – 3(0) – 1 .... equ .39

$[MT^{-1}]^{1}$= $[\mathrm{M}]^{-1}[\mathrm{T}]^{1}[ML^{2}T^{-3}]^{0}[\mathrm{L}]^{0}[\mathrm{L}]^{0}[ML^{-1}T^{-1}]^{0}$ .... equ .40

$\pi_{11} = \frac{T_p\, x\, T_t}{M}$ .... equ .41

Assume arbitrary values of a =1, b = 0, f = 0, g = 1, to obtain π_{22}:

M = (0) + (b) + 1 = 0 .... equ .42

L= 2(0) + e + f – g = 0 .... equ .43

T = -(0) + c – 3(0) – 1 = 0 .... equ .44

$\pi_{22} = \frac{P\, x\, T_t}{M}$ .... equ .45

$[MT^{-1}]^{1}$= $[\mathrm{M}]^{-1}[\mathrm{T}]^{1}[ML^{2}T^{-3}]^{1}[\mathrm{L}]^{1}[\mathrm{L}]^{1}[ML^{-1}T^{-1}]^{0}$ .... equ .44

Assume arbitrary values of a =1, c = 0, d = 0, e = 1, g = 0 to obtain π_{33}:

M = (1) + (0) + 0 = 0 .... equ .45

L= 2(0) + 1 + f – 0 = 0 .... equ .46

T = -(0) + 0 – 3(0) – 1 = 0 .... equ .47

$\pi_{33} = \frac{D}{S}$ .... equ .48

$[MT^{-1}]^{0}$= $[\mathrm{M}]^{0}[\mathrm{T}]^{0}[ML^{2}T^{-3}]^{1}[\mathrm{L}]^{1}[\mathrm{L}]^{-1}[ML^{-1}T^{-1}]^{0}$ .... equ .49

Assume arbitrary values of a = 0, d = 0, d = 0, e = 0, g = 1 to obtain π_{44}:

M = (1) + (0) + 0 = 0 .... equ .50

L= 2(0) + 1 + f – 0 = 0 .... equ .51

T = -(0) + c – 3(0) – 1 = 0 .... equ .52

$\pi_{44} = \frac{\mu\, x\, T_t}{M}$ .... equ .53

$[MT^{-1}]^{0}$= $[\mathrm{M}]^{-1}[\mathrm{T}]^{1}[ML^{2}T^{-3}]^{0}[\mathrm{L}]^{0}[\mathrm{L}]^{0}[ML^{-1}T^{-1}]^{1}$ .... equ .54

[Shafii, 1996] suggested that the process of making dimension terms more manageable by reducing them can be accomplished through the utilization of multiplication and division. However, we have the equation 55

$\boldsymbol{\pi_{55} = \frac{\pi_{44}\, x\, \pi_{33}}{\pi_{22}}}$ .... equ .55

$\boldsymbol{\pi_{55} = \frac{\mu\, x\, M\, x\, D}{S\, x\, P}}$ .... equ .56

In order to determine the T_p-model, equate equation. 56 into equation 33. However, the throughput model can be expressed as:

$\frac{T_p \, x \, T_t}{M} = \frac{\mu \, x \, M \, x \, D}{S \, x \, P}$ .... equ .57

$T_p = C_t \left(\frac{\mu \, x \, M^2 \, x \, D}{S \, x \, P \, x \, T_t}\right)$ .... equ .58

3.3 Value of the Constant

3.3.1 Determination of the efficiency constant

The determination of the efficiency capacity C_e involved the utilization of linearized expressions for π4 and π1 (namely, $\left[\frac{t_B{}^2 N\theta}{V_B} \; and \; \eta\right]$ extracted from the developed model. This process was executed employing the method of least squares, a statistical technique, as elaborated by the works of [Bolaji, 2008] and [Ikejiofor, 2016]. Consequently, the gradient of the line that best fits the data furnished the value of the efficiency constant, which, intriguingly, was ascertained to be 1E-06. However, as shown in Figure 1, the regression coefficient, R, obtained was 0.8782.

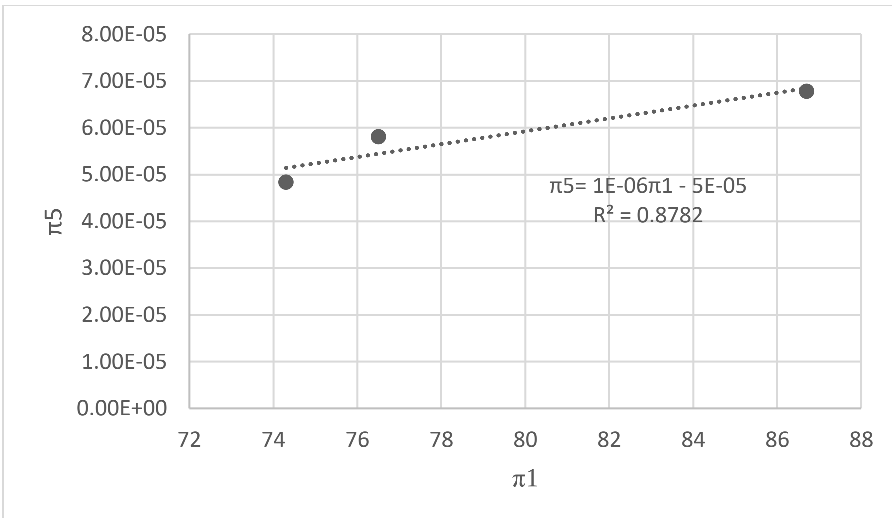

Figure 1 Determination of efficiency constant, C_e

3.3.2 Determination of the throughput constant

Determining the throughput capacity C_t involved using simplified equations for π44 and π11, specifically $\left[\frac{\mu \, x \, T_t}{M} \; and \; \frac{T_p \, x \, T_t}{M}\right]$. These equations were derived from a complex model we created. We used a reliable statistical technique called the method of least squares, supported by the research of [Bolaji, 2008] and [Ikejiofor, 2016]. By analysing the data and finding the best-fitting line, we discovered an interesting result for the throughput constant. This value, 9E-05, explains the efficiency of the process in terms of throughput. Thus, as shown in Figure 2, the regression coefficient, R, 0.983, was obtained.

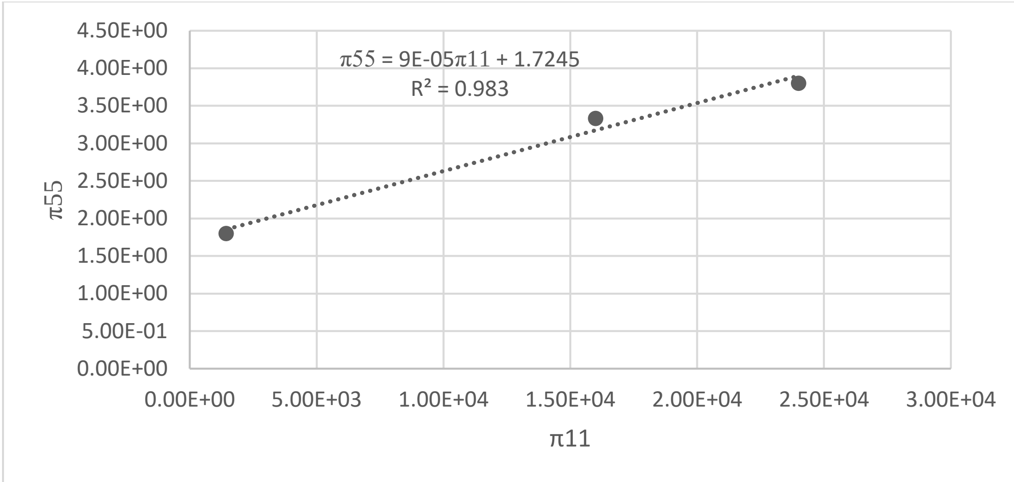

Figure 2 Determination of efficiency constant, C_t

4. Model Validation

4.1 Efficiency Model Validation

In this experiment, six process parameters were employed, including the density of the palm fruit, volume of the digester drum, blade angle, number of blades, machine torque, and blade thickness. The efficiency model was verified to confirm its consistency with established experimental results from the previous study [Erhimona, 2023]. Thus, the developed palm fruit digester machine and the predicted model were compared with the experimental result. The operating conditions for the palm fruit piece and the digester machine used are presented in Table 5.

Table 5 Operating conditions for the palm fruit piece and the digester

S/N	Parameter	Levels		
		1	**2**	**3**
1	Density of the palm fruit piece, ρ ($\mathrm{Kgm^{-3}}$)	750	800	950
2	The volume of the digester, $V_D(\mathrm{m^3})$	95	95	95
3	Blade angle, θ	37	37	37
4	Number of blades, N	5	6	7
5	Machine torque, T_m (N-m)	16	17	18
6	Blade thickness, t_B (mm)	5	5	5

A model gains more trustworthiness when it fits the data well and is statistically meaningful. To gauge this, we used a method outlined by [Fedler, 1986]. This method calculates something called the coefficient of determination, often referred to as R^2. This number tells us how well the model works, and it's useful for both linear and non-linear functions. It's versatile enough to handle cases with one or more independent variables. Using this method adds weight to our findings and analysis. Thus,

$$t = \frac{R(Df)^{\frac{1}{2}}}{(1-R^2)^{\frac{1}{2}}}$$

where:

t = student's t – value

R = coefficient of regression

Df= degrees of freedom (number of data points minus the number of constants as defined in the model)

4.2 Throughput Model Validation

For this experiment, we considered six specific factors: the mass of boiled palm fruit, digestion time, machine power, drum diameter, size of the palm fruit pieces, and the viscosity of the extracted palm oil. We tested our throughput model to ensure it matched the outcomes of an earlier study [Erhimona, 2023]. This validation was important to confirm that our model is in line with established experimental findings. To assess our results, we compared the predictions made by our developed palm fruit digester machine with the actual experimental data. The details of the conditions under which the palm fruit pieces and the digester machine operated can be found in Table 6.

Table 6: Operating conditions for the palm fruit piece and the digester

S/N	Parameter	Levels		
		1	**2**	**3**
1	Mass of boiled palm fruit, M (Kg)	8	9	10
2	Digestion time, T_t (sec)	120	210	300
3	Machine Power, P (watt)	2	2	2
4	Drum diameter, D (mm)	450	450	450
5	Size of the palm fruit pieces, S (mm)	3	4	5
6	Viscosity of the extracted palm oil μ (Pa.sec)	35.62	35.62	35.62

A model becomes more reliable when it fits the data well and has statistical importance. To understand this, we used a method from [Fedler, 1986]. This method calculates a thing called the coefficient of determination, often called R^2. This number helps us see how well the model is doing, and it works for both straight and curvy relationships. It can even handle situations with more than one independent thing that affects the outcome.

5. Results

In Table 7, you can see the machine efficiencies that were both calculated and measured. Equation 30 was used to calculate the measured efficiency values. The outcome of this analysis showed that the model aligned well with the measured data from the developed palm fruit digester. The value of R^2, which helps us understand how well the model fits the data, was found to be 0.9956. Table 7 and Figure 3 present the data used to compare the efficiency obtained from experimental results and those derived from the performance models we formulated. When we compared the average predicted and measured throughput capacities, a statistical analysis showed that there wasn't a significant difference between the averages at a 5% level of significance. This was indicated by the calculated 't' value (0.9952) being smaller than the table 't' value (1.0203). These findings highlight a strong relationship between the model and the data obtained from the developed palm fruit digester.

Table 7 The Measured and computed efficiencies for the digester machine

Number of blades N	Efficiency (%)	
	Computed	Measured
5	74.2	74.3
6	75.5	76.5
7	84.7	86.7

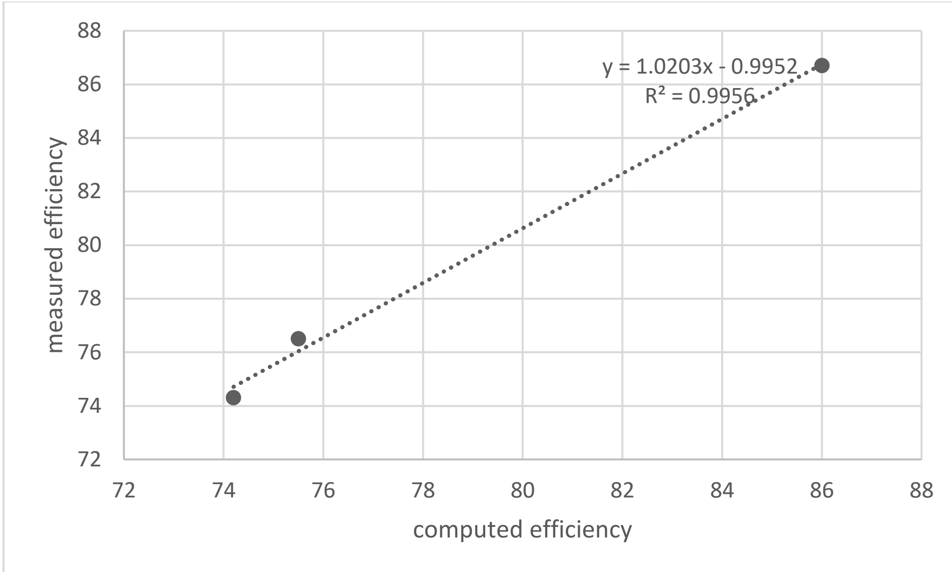

Figure 3 Measured efficiency versus computed efficiency

Table 8 provides a glimpse of both the calculated and measured machine throughput. We utilized Equation 58 to examine the calculated efficiency values. The results of this analysis demonstrated that the model corresponded well with the actual data collected from the developed palm fruit digester machine. The value of R^2, which gives us insight into how accurately the model fits the data, was determined to be 0.9956. Both Table 8 and Figure 4 exhibit the data that was used to compare the throughput capacity values obtained through experimental means and those generated from the performance models we established. When we evaluated the average predicted and measured throughput capacities, a statistical assessment revealed that there wasn't a significant difference in the averages at a 5% level of significance. This conclusion was supported by the calculated 't' value (0.9956) being lower than the table 't' value (1.0203). These discoveries underscore a robust connection between the model and the data derived from the developed palm fruit digester.

Table 8 The Measured and computed throughput values for the digester machine

Mass of the boiled palm fruit Kg	Throughput (Kg/sec) Computed	Measured
8	0.12	0.10
9	0.113	0.114
10	0.145	0.145

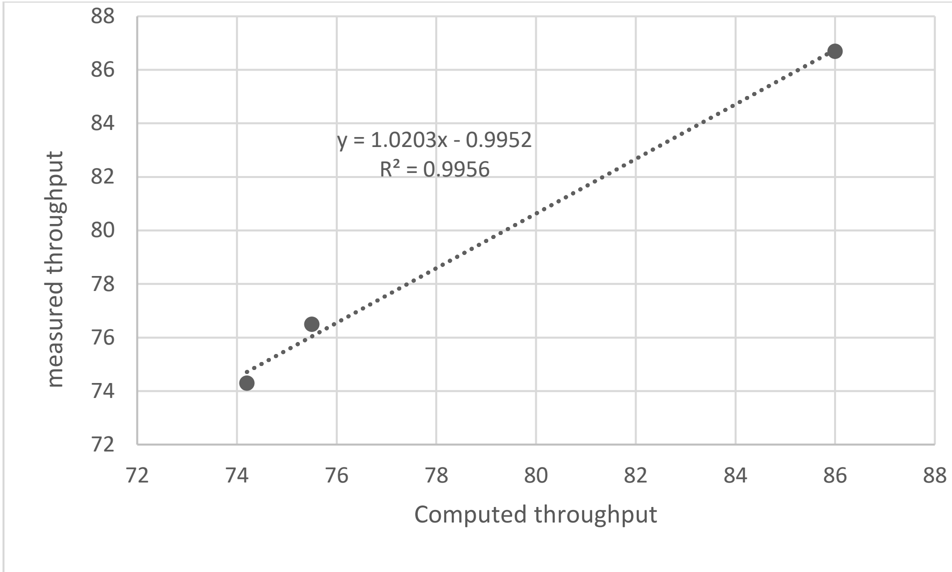

Figure 4 Measured throughput versus computed throughput

6. Conclusion

A mathematical framework was introduced to predict the palm fruit digester machine's efficiency and capacity throughput. This framework utilizes dimensional analysis and is founded on Buckingham's pi theorem. The model equation, represented as $T_p = C_t\left(\frac{\mu \, x \, M^2 \, x \, D}{S \, x \, P \, x \, T_t}\right)$ and $\eta = C_e\left(\frac{t_B{}^2 N\theta}{V_B}\right)$ for predicting the throughput and machine efficiency, respectively, was confirmed through the use of data from the palm fruit digester machine in earlier studies [Onyenanu, 2023] [Erhimona, 2023]. The R^2 value obtained from the analysis were 0.9956 and 0.9956 for the throughput and machine efficiency, respectively. Thus, the model can predict the efficiency and capacity throughput of the palm fruit digester.

6.1. Limitations and Further Recommendations

Data limitation, simplifying assumptions, model calibration, and validation are some of the difficulties that come with creating a mathematical model for palm fruit digester design. Although the combination of process optimization and dimensional analysis yields insightful information, recognizing the limitations of the study helps put the findings in a realistic perspective. To improve the applicability and reliability of findings in real-world scenarios, future research should address these limitations, possibly by expanding validation efforts, improving data collection techniques, and fine-tuning model assumptions.

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