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| RESEARCH ARTICLE

An Exponential Ratio Type Estimator of Population Mean in Presence of Measurement Error and Non-response Error

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| ABSTRACT

In this paper, we proposed an exponential ratio type estimator for estimating the population mean in the presence of measurement error and non-response error. Also, assume that these two errors are present in both the study variable and the auxiliary variable. The theoretical expression of bias and mean square error are also derived up to the first order of approximation for the proposed estimator. A numerical study has been worked out to prove the superiority of the proposed estimator in comparison of other existing estimators using an artificially generated bivariate data set.

| KEYWORDS

Efficiency, Non-response, Measurement Error, Mean Squared Error, Mean Estimation.

| ARTICLE INFORMATION

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1. Introduction

In survey sampling, accurate estimation of population parameters is paramount for informed decision-making. However, real-world surveys often face challenges such as measurement error and non-response error, which can significantly distort the accuracy of estimates. To address these challenges, researchers have developed innovative estimation techniques.

To tackle the issue of non-response, Hansen and Hurwitz (1946) proposed a method for estimating the population mean. This method involves sending a questionnaire to all units in the sample by mail and then selecting a sub-sample from the units that did not respond. A direct or telephone interview is conducted with the non-respondent sub-sample, assuming that every unit in the sub-sample will respond. Various researchers have proposed estimators over the decades, such as Cochran (1977), Rao (1986), Okafor and Lee (2000), Kreuter et al. (2010), Khan et al. (2014), Luengo (2016), Khare and Sinha (2019), Sharma and Kumar (2020), Pandey et al. (2021), and Sinha et al. (2022).

Measurement inaccuracy has an impact on real-world survey accuracy in addition to non-response. Although it is believed that all data that is processed and recorded is accurate, this isn't always the case in real-world situations. Many variables, including respondent and interviewer bias and mistakes in data processing and recording, can lead to measurement inaccuracy. The problem of estimating in the presence of measurement error has been studied by a number of researchers, including Fullar (1987), Hausman (2001), Srivastava and Shalabh (2001), Brown and Mathiowetz (2001), Singh and Karpe (2008,2009), Kumar et al (2011).

Considering that non-response and measurement error are commonly encountered in surveys, it is important to study both of these factors simultaneously. However, there have been limited studies in this area. Kumar et al., Singh and Sharma (2015), Azeem and Hanif (2016), Singh and Singh (2018), Kumar and Bhougal (2015), Tiwari (2023). have made contributions in this field.

Therefore, in this article we focus on estimating the population mean efficiently while accounting for both non-response and measurement error. Proposed estimator offers a flexible framework that accommodates the complexities introduced by measurement error and non-response error. These estimators leverage the inherent relationships between auxiliary variables and the variable of interest to improve estimation accuracy. By exploiting the correlation between auxiliary variables and the target variable, these estimators can mitigate the impact of errors, leading to more reliable population estimates.

The structure of the current paper is: Notations are present in section 2. Some current estimators of the finite population mean in presence of non-response and measurement error are provided in Section 3. In Section 4, a better class of estimators that simultaneously incorporate measurement error and non-response data is proposed for estimating the finite population mean. Section 5 presents a numerical study. Section 6 provide the result. Conclusion present in section 7.

2. Notations

Consider a finite population of size N, let this population be divided into two strata, respondent and nonrespondents of size N_1 and N_2 respectably, a sample of size n is drawn randomly from this population, among which n_1 are respondent and n_2 are nonrespondent units. Further, a sub sample of size $r\left(=\frac{n_2}{k}\right), k>0$ is taken from n_2 non respondent units. Let x_i and y_i be the observed values for $i^{th}(i=1,2,3,\ldots,n)$ selected unit.

The population mean and variance for the study variable y are $\mu_y=\frac{1}{N}\sum_{i=1}^{N}Y_i$, $S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}(Y_i-\bar{Y})^2$.

The population mean and variance for the auxiliary variable x are $\mu_x=\frac{1}{N}\sum_{i=1}^{N}X_i$, $S_X^2=\frac{1}{N-1}\sum_{i=1}^{N}(X_i-\bar{X})^2$.

Population variance for y and x for the group of non-respondents is $S_{Y(2)}^2=\frac{1}{N_2-1}\sum_{i=1}^{N_2}(Y_i-\bar{Y})^2$and $S_{X(2)}^2=\frac{1}{N_2-1}\sum_{i=1}^{N_2}(X_i-\bar{X})^2$ respectively.

Hansen and Hurwitz (1946) defined the estimator of the population mean in the presence of non-response as:

$$\bar{y}_t^*=w_1\bar{y}_{n_1}+w_2\bar{y}_r$$

where $w_1=\left(\frac{n_1}{n}\right)$, $w_2=\left(\frac{n_2}{n}\right)$, $and\ \bar{y}_{n_1}=\frac{1}{n_1}\sum_{i=1}^{n_1}y_i$, $\bar{y}_r=\frac{1}{r}\sum_{i=1}^{r}y_i$.

The variance $\bar{y}^*$ is given as

$$V(\bar{y}^*)=\bar{Y}^2\left[\lambda C_y^2+\theta C_{y(2)}^2\right]$$

where $\lambda=\left(\frac{1-f}{n}\right)$, $\theta=\frac{W_2\,(K-1)}{n}$, $C_y^2=\frac{S_y^2}{\mu_y^2}$ and $C_{y(2)}^2=\frac{S_{y(2)}^2}{\mu_y^2}$.

The similar expression will be used for the auxiliary variable x.

Let X_i, Y_i be the true value for the $i^{th}(i=1,2,3,\ldots,N)$ unit in the population and $u_i=y_i-Y_i$, $v_i=x_i-X_i$ be the measurement error on the study and the auxiliary variable respectively with mean zero and the population variance for study variable Y is $S_U^2=\frac{1}{N-1}\sum_{i=1}^{N}(U_i-\bar{U})^2$and population variance for auxiliary variable X is $S_V^2=\frac{1}{N-1}\sum_{i=1}^{N}(V_i-\bar{V})^2$, let the $u_i's$ and $v_i's$ are uncorrelated but $X_i's$ and $Y_i's$ are correlated and ρ be the population correlation coefficient between x and y.

The usual unbiased estimator for the population mean of the study variable in the presence of measurement error is given as

$$\bar{y}^* = \frac{1}{n}\sum_{\{i=1\}}^{n} y_i$$

The variance of $\bar{y}^*$ is given as

$$Var(\bar{y}^*) = \left(\frac{1}{n} - \frac{1}{N}\right)(S_y^2 + S_U^2)$$

In the presence of measurement error the population variance of U and V for the non respondent group are $S_{U_{(2)}}^2 = \frac{1}{N_2-1}\sum_{i=1}^{N_2}(U_i - \bar{U})^2$ and $S_{V_{(2)}}^2 = \frac{1}{N_2-1}\sum_{i=1}^{N_2}(V_i - \bar{V})^2$ respectively.

So, if we denote the unbiased estimator of population mean in presence of measurement error and non-response both as $\bar{y}^{**}$, given as

$$\bar{y}_t^{**} = w_1\bar{y}_{n_1}^* + w_2\bar{y}_r^*$$

and

$$V(\bar{y}^{**}) = \lambda S_y^2\left(1 + \frac{S_u^2}{S_y^2}\right) + \theta S_{y_2}^2\left(1 + \frac{S_{u_{(2)}}^2}{S_{y_{(2)}}^2}\right)$$

The similar expression will be used for the auxiliary variable X.

We define, $\bar{y}^{**} = \mu_y(1 + e_0)$, $\bar{x}^{**} = \mu_x(1 + e_1)$, so that $E(e_0) = 0 = E(e_1)$, and set the following notations for deriving the bias an mean square error of proposed and existing estimators.

$$E(e_o^2) = \lambda C_y^2\left(1 + \frac{S_u^2}{S_y^2}\right) + \theta C_{y_2}^2\left(1 + \frac{S_{u_2}^2}{S_{y_2}^2}\right) = M(say)$$

$$E(e_1^2) = \lambda\left(1 + \frac{S_v^2}{S_x^2}\right) + \theta C_{x_2}^2\left(1 + \frac{S_{v_2}^2}{S_{x_2}^2}\right) = N(say)$$

$$E(e_o e_1) = \lambda\,\rho_{yx}C_yC_x + \theta\rho_{yx_2}C_{y_2}C_{x_2} = O(say)$$

where, $\rho_{yx} = \frac{s_{yx}}{s_y s_x}$, $R = \mu_y/\mu_x$,

3. Existing and Adapted of Estimators of Population Mean in presence of Non response and Measurement Error

This section discusses a few existing and adapted estimators in presence of non-response and measurement error for estimating population mean that will be employed going forward from the literature.

A traditional unbiased estimator for estimating population mean in the simultaneous presence of and non-response and measurement error is given by

$$t_1 = \frac{1}{n}\sum_{i=1}^{n} y_i = \bar{y}^{**} \tag{1}$$

The mean square error of t_1 is given as

$$MSE\,(t_1) = \mu_y{}^2\left[\lambda C_y^2\left(1 + \frac{S_u^2}{S_y^2}\right) + \theta\,C_{y_2}^2\left(1 + \frac{S_{u_2}^2}{S_{y_2}^2}\right)\right] = \mu_y^2\,M \tag{2}$$

Using Searls (1964) constant, the estimator of the population mean $\bar{Y}$ is given as

$$t_2 = K\bar{y}^{**} \tag{3}$$

where the optimum value of K is obtained by minimizing $MSE(t_2)$

$$K = 1/(1 + M)$$

The Bias and mean square error of t_2 is given as,

$$Bias(\,t_2) = (K-1)\,\mu_y$$

$$\min MSE\,(t_2) = \frac{\mu_y^2 M}{(1+M)} \qquad (4)$$

The ratio estimator, Cochran (1940), in the presence of measurement error and non- response error for estimating the population mean $\bar{Y}$ is given as

$$t_3 = \bar{y}^{**}\left(\frac{\mu_x}{\bar{x}^{**}}\right) \qquad (5)$$

The Bias and Mean Square Error of t_3 is given as

$$Bias(t_3) = \mu_y(N-O)$$

$$MSE(t_3) = \mu_y^2(M+N-2O) \qquad (6)$$

Murthy (1964) product estimator of population mean in simultaneous presence of non-response and measurement error is given as

$$t_4 = \bar{y}^{**}\left(\frac{\bar{x}^{**}}{\mu_x}\right) \qquad (7)$$

The Bias and Mean Square Error of t_4 is given as,

$$Bias(t_4) = \mu_y\,O$$

$$MSE(t_4) = \mu_y^2(M+N+2O) \qquad (8)$$

The regression estimator of population mean $\bar{Y}$ in presence of measurement error and non-response is given as

$$t_5 = \bar{y}^{**} + b_{yx}(\mu_x - \bar{x}^{**}) \qquad (9)$$

The optimum value of b_{yx} is obtained by minimizing MSE, which is given as

$$b_{yx} = R\frac{O}{N}$$

$$MSE_{min}\,(t_5) = \mu_y^2\left[M - \frac{O^2}{N}\right] \qquad (10)$$

Srivastava (1967) ratio estimator of population mean $\bar{Y}$ in simultaneous presence of non-response and measurement error is given as

$$t_6 = \bar{y}^{**}\left(\frac{\mu_x}{\bar{x}^{**}}\right)^{\alpha_2} \qquad (11)$$

The optimum value of α_2 is obtained by minimizing MSE, which is given as

$$\alpha_2 = \frac{O}{N}$$

The Bias and Mean Square Error of t_5 is given as,

$$Bise(t_6) = \mu_y\left\{\frac{\alpha_2(\alpha_2-1)}{2}\,N - \alpha_2 O\right\}$$

$$MSE_{min}(t_6) = \mu_y^2\left(M - \frac{O^2}{N}\right) \qquad (12)$$

Bahl and Tuteja (1991) ratio and product type estimator of population mean in the presence non-response and measurement error is given as.

$$t_7 = \bar{y}^{**} \exp\left(\frac{\mu_x - \bar{x}^{**}}{\mu_x + \bar{x}^{**}}\right) \qquad (13)$$

$$t_8 = \bar{y}^{**} \exp\left(\frac{\bar{x}^{**} - \mu_x}{\bar{x}^{**} + \mu_x}\right) \qquad (14)$$

The Bias and Mean Square Error of t_7 is given as,

$$Bias(t_7) = \mu_y\left(\frac{3N}{8} - \frac{O}{2}\right)\ \mu_y$$

$$MSE(t_7) = \mu_y^2\left(M + \frac{N}{4} - O\right) \qquad (15)$$

The Bias and Mean Square Error of t_8 is given as,

$$Bias(t_8) = \mu_y\left(\frac{O}{2} - \frac{N}{8}\right)$$

$$MSE(\ t_8) = \mu_y^2\left(M + \frac{N}{4} + O\right) \qquad (16)$$

Singh and Espejo (2003) estimator of population mean $\bar{Y}$ in the simultaneous presence of non-response and measurement error is given as

$$t_9 = \bar{y}^{**}\left\{a\left(\frac{\mu_x}{\bar{x}^{**}}\right) + (1-a)\left(\frac{\bar{x}^{**}}{\mu_x}\right)\right\} \qquad (17)$$

The optimum value of a is obtained by minimizing MSE, which is given as

$$a = \frac{N + O}{2N}$$

The Bias and Mean Square Error of t_9 is given as,

$$Bias(t_9) = \mu_y\{O(1-2a) + aN\}$$

$$MSE_{min}(t_9) = {\mu_y}^2\left(M - \frac{O^2}{N}\right) \qquad (18)$$

4. Proposed Estimator of Population mean in presence of Non response and Measurement error:

In the present study, motivated by Tiwari et al. (2022), we propose a general class of new exponential ratio and difference type estimators for estimating the population mean of a study variable Y in the presence of non-response and measurement error, utilizing information available on an auxiliary variable X. Measurement errors are considered in both the variables Y and X. The proposed class of estimator is defined as follows:

$$t_{10} = \left(k_1\,\bar{y}^{**} + k_2(\mu_x - \bar{x}^{**})\right)\left(\frac{\mu_x}{\bar{x}^{**}}\right)^{\alpha}\ exp\left\{\frac{\delta(\mu_x - \bar{x}^{**})}{(\mu_x + \bar{x}^{**})}\right\} \qquad (19)$$

Here α and δ is a given constant and k_1 and k_2 are chosen by minimizing MSE.

The proposed class of estimator t_{10} is reduced to some known estimators by putting different values of constants $(\mathrm{k}_1, \mathrm{k}_2, \delta, \alpha)$ as shown in Table 1.

Table 1: Some members of the proposed class of estimator

	$\boldsymbol{k_1}$	$\boldsymbol{k_2}$	$\boldsymbol{\alpha}$	$\boldsymbol{\delta}$	$\hat{\boldsymbol{\mu}}_{\Delta id}$
1.	1	0	0	0	$t_1 = \bar{y}^{**}$
2.	k	0	0	0	$t_2 = k\bar{y}^{**}$
3.	1	0	1	0	$t_3 = \bar{y}^{**}\left(\frac{\mu_x}{\bar{x}^{**}}\right)$
4.	1	0	-1	0	$t_4 = \bar{y}^{**}\left(\frac{\bar{x}^{**}}{\mu_x}\right)$
5.	1	β_{zx}	0	0	$t_5 = \bar{y}^{**} + b_{yx}(\mu_x - \bar{x}^{**})$
6.	1	d	0	0	$t_5 = \bar{y}^{**} + d(\mu_x - \bar{x}^{**})$
7.	1	k	α	0	$t_6 = \bar{y}^{**}\left(\frac{\mu_x}{\bar{x}^{**}}\right)^{\alpha}$
8.	1	0	0	1	$t_7 = \bar{y}^{**}\exp\left(\frac{\mu_x - \bar{x}^{**}}{\mu_x + \bar{x}^{**}}\right)$
9.	1	0	0	-1	$t_8 = \bar{y}^{**}\exp\left(\frac{\bar{x}^{**} - \mu_x}{\bar{x}^{**} + \mu_x}\right)$
10.	k_1	k_2	0	δ	$\hat{\mu}_{\Delta 5d} = [k_1\bar{z} + k_2(\mu_x - \bar{x}^{**})]\exp\left\{\frac{\delta(\mu_x - \bar{x}^{**})}{(\mu_x + \bar{x}^{**})}\right\}$

4.1 Bias and MSE of the proposed estimator:

Equation (19) in terms of e's and ignoring the terms of order (n^{-2}), we can write

$$t_{10} = \mu_y\left[k_1 + k_1e_0 - \left(\frac{k_2}{R} + \alpha k_1 + \frac{k_1\delta}{2}\right)e_1 + \left(\frac{k_2\alpha}{R} + k_1\frac{\alpha(\alpha+1)}{2} + \frac{k_2}{R}\frac{\delta}{2} + k_1\alpha\frac{\delta}{2} + \frac{k_1\delta}{4} + \frac{k_1\delta^2}{8}\right)e_1^2 - \left(k_1\alpha + \frac{k_1\delta}{2}\right)e_0e_1\right] \qquad (20)$$

Subtracting μ_y both sides from equation (20)

$$t_{10} - \mu_y \cong \mu_y\left[(k_1 - 1) + k_1e_0 - \left(\frac{k_2}{R} + \left(\alpha + \frac{\delta}{2}\right)k_1\right)e_1 + \left(k_2\left(\frac{\alpha}{R} + \frac{\delta}{2R}\right) + k_1\left(\frac{\alpha(\alpha+1)}{2} + \alpha\frac{\delta}{2} + \frac{\delta}{4} + \frac{\delta^2}{8}\right)\right)e_1^2 - \left(\alpha + \frac{\delta}{2}\right)k_1e_0e_1\right] \qquad (21)$$

Taking expectation on both sides of equation (21)

$$Bias(t_{10}) = \mu_y\left[(k_1 - 1) + \left(k_2\left(\frac{\alpha}{R} + \frac{\delta}{2R}\right) + k_1\left(\frac{\alpha(\alpha+1)}{2} + \alpha\frac{\delta}{2} + \frac{\delta}{4} + \frac{\delta^2}{8}\right)\right)N - \left(\alpha + \frac{\delta}{2}\right)k_1O\right] \qquad (22)$$

Squaring both sides of equation (21)

$$\left(t_{10} - \mu_y\right)^2 = \mu_y^2\left((k_1 - 1)^2 + k_1^2e_0^2 + \left(\frac{k_2^2}{R^2} + \left(\alpha + \frac{\delta}{2}\right)^2k_1^2 + 2\frac{1}{R}\left(\alpha + \frac{\delta}{2}\right)k_1k_2\right)e_1^2 + 2\left(\frac{k_1k_2\alpha}{R} + k_1^2\frac{\alpha(\alpha+1)}{2} + \frac{k_1k_2}{R}\frac{\delta}{2} + k_1^2\alpha\frac{\delta}{2} + \frac{k_1^2\delta}{4} + \frac{k_1^2\delta^2}{8} - \frac{k_2\alpha}{R} - k_1\frac{\alpha(\alpha+1)}{2} - \frac{k_2}{R}\frac{\delta}{2} - k_1\alpha\frac{\delta}{2} - \frac{k_1\delta}{4} - \frac{k_1\delta^2}{8}\right)e_1^2 - 2\left(k_1^2\alpha + \frac{k_1^2\delta}{2} - k_1\alpha - \frac{k_1\delta}{2}\right)e_0e_1 - 2\left(\frac{k_1k_2}{R} + \left(\alpha + \frac{\delta}{2}\right)k_1^2\right)e_0e_1\right)$$

(23) Taking expectation on both side of (23) we get the mean square error of the proposed estimator

$$MSE(t_{10}) = E\left(t_{10} - \mu_y\right)^2 = \mu_y^2\left[1 + k_1^2\left\{1 + M + \left(2\alpha^2 + \alpha + \frac{\delta^2}{2} + 2\alpha\delta + \frac{\delta}{2}\right)N - 4\left(\alpha + \frac{\delta}{2}\right)O\right\} + \frac{k_2^2}{R^2}N - 2k_1\left\{1 + \left(\frac{\alpha(\alpha+1)}{2} + \alpha\frac{\delta}{2} + \frac{\delta}{4} + \frac{\delta^2}{8}\right)N - \left(\alpha + \frac{\delta}{2}\right)O\right\} - 2k_2\left\{\left(\frac{\alpha}{R} + \frac{1}{R}\frac{\delta}{2}\right)N\right\} + 2k_1k_2\left\{\left(\frac{1}{R}\left(\alpha + \frac{\delta}{2}\right)\right) + \frac{\alpha}{R} + \frac{\delta}{2R}\right\}N - \frac{O}{R}\right]$$

(24) $= \mu_y^2(1 + k_1^2P_1 + k_2^2P_2 - 2k_1P_4 -$
$2k_2P_5 + 2k_1k_2P_3$ (25)
where,

$$P_1 = 1 + M + \left(2\alpha^2 + \alpha + \frac{\delta^2}{2} + 2\alpha\delta + \frac{\delta}{2}\right)N - 4\left(\alpha + \frac{\delta}{2}\right)O$$

$$P_2 = \frac{1}{R^2}N$$

$$P_3 = \left(\left(\frac{1}{R}\left(\alpha + \frac{\delta}{2}\right)\right) + \frac{\alpha}{R} + \frac{\delta}{2R}\right)N - \frac{O}{R}$$

$$P_4 = 1 + \left(\frac{\alpha(\alpha+1)}{2} + \alpha\frac{\delta}{2} + \frac{\delta}{4} + \frac{\delta^2}{8}\right)N - \left(\alpha + \frac{\delta}{2}\right)O$$

$$P_5 = \left(\frac{\alpha}{R} + \frac{1}{R}\frac{\delta}{2}\right)N$$

4.2 Optimum values of k_1 and k_2 and the Minimum Mean Square error

Optimum values of k_1 and k_2 are obtained by minimizing the mean square error of the proposed estimator t_{10} given in equation (25). To get the optimum values of k_1 and k_2 , we minimize the equation (25) with respect to k_1 and k_2 respectively and equate them to zero, i.e. by setting

$$\frac{\partial MSE(t10\ )}{\partial k_i} = 0 \text{ for } i = 1, 2$$

We get

$$k_{1opt} = \frac{P_4P_2 - P_3P_5}{P_2P_1 - P_3^2}$$

and

$$k_{2opt} = \frac{P_5P_1 - P_4P_3}{P_2P_1 - P_3^2}$$

The Minimum mean square error of t_{10}, is now given as after putting the value of k_{1opt} and k_{2opt} is

$$MSE_{min}(\hat{\mu}_{\Delta id}) = \mu_y^2\left[1 - \frac{(P_4^2P_2 + P_1P_5^2 - 2P_3P_4P_5)}{(P_2P_1 - P_3^2)}\right]$$

5. Numerical Study

To evaluate the efficiency of our proposed estimator with respect to the other existing and adapted estimator, the Mean square error and Percentage relative efficiency of these estimators are calculated for an artificially generated bivariate date set (Azeem, M. and Hanif, M. (2015). using MVNORM function in the MASS package of R-software, the true auxiliary variable is assumed as $X \sim N(10,2)$ and the measured auxiliary variable is assumed as $x = X + N(0,1)$ then $V = x - X$. The study variable is then generated by a linear model $Y = 2 + bX + N(0,1)$. The values of b for numerical investigation are taken as$0.1, -0.1, 0.3,$ and -0.3. The measured study variable is generated as $y = Y + N(0,1)$ then $= y - Y$.

A population of size 5000 is generated and a sample of size 500 is drawn randomly from it. Considering the respondents subpopulation (N_1) is 3750. It is assumed that 70% of the sample are responders, and for the additional re-contacting sample size we considered three cases of sub sampling ratio i. e.$k = 2,3,4$.

The mean square errors (MSEs) and percent relative efficiencies (PREs) are calculated for the methods considered in the article with respect to the mean estimator (t_1). The PRE of the estimators are calculated using the formula

$$PRE = \frac{MSE(t_1)}{MSE(t_i)} \times 100\%, \quad i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.$$

The results are tabulated for various values of α and δ are presented in Table 1 to Table 4 for populations generated according to $b = -0.3, 0.3, -0.1,\ and\ 0.1$.

Table 2: - This table shows PRE for $b = -0.3$

Estimators	**k=2**	**k=3**	**k=4**
t_1	100.00000	100.00000	100.00000
t_2	100.9256	101.12121	101.31696
t_3	94.11559	94.13205	94.14361
t_4	70.66365	70.70599	70.73577
$t_5 = t_6 = t_9$	198.89079	198.73731	198.62988
t_7	98.46098	98.46548	98.46864
t_8	83.89638	83.92294	83.94162
$t_{10}(\alpha = -2)$			
$\delta = -2$	**198.9509**	**198.8119**	**198.7190**
$\delta = -1$	**199.0842**	**198.9739**	**198.9098**
$\delta = 0$	**199.2556**	**199.1816**	**199.1536**
$\delta = 1$	**199.4358**	**199.3994**	**199.4090**
$\delta = 2$	**199.6009**	**199.5986**	**199.6424**
$t_{10}(\alpha = -1)$			
$\delta = -2$	**199.2556**	**199.1816**	**199.1536**
$\delta = -1$	**199.4358**	**199.3994**	**199.4090**
$\delta = 0$	**199.6009**	**199.5986**	**199.6424**
$\delta = 1$	**199.7321**	**199.7570**	**199.8279**
$\delta = 2$	**199.8162**	**199.8585**	**199.9468**
$t_{10}(\alpha = 0)$			
$\delta = -2$	**199.6009**	**199.5986**	**199.6424**
$\delta = -1$	**199.7321**	**199.7570**	**199.8279**
$\delta = 0$	**199.8162**	**199.8585**	**199.9468**
$\delta = 1$	**199.8454**	**199.8937**	**199.9881**
$\delta = 2$	**199.8168**	**199.8593**	**199.9479**
$t_{10}(\alpha = 1)$			
$\delta = -2$	**199.8162**	**199.8585**	**199.9468**
$\delta = -1$	**199.8454**	**199.8937**	**199.9881**
$\delta = 0$	**199.8168**	**199.8593**	**199.9479**
$\delta = 1$	**199.7330**	**199.7584**	**199.8299**
$\delta = 2$	**199.6021**	**199.6004**	**199.6449**
$t_{10}(\alpha = 2)$			
$\delta = -2$	**199.8168**	**199.8593**	**199.9479**
$\delta = -1$	**199.7330**	**199.7584**	**199.8299**
$\delta = 0$	**199.6021**	**199.6004**	**199.6449**
$\delta = 1$	**199.4371**	**199.4012**	**199.4116**
$\delta = 2$	**199.2567**	**199.1831**	**199.1558**

Table 3: - This table shows PRE for $b = 0.3$

Estimators	$k = 2$	$k = 3$	$k = 4$
t_1	100.00000	100.00000	100.00000
t_2	100.03874	100.04689	100.05504
t_3	40.14004	40.21281	40.26418
t_4	23.52684	23.57721	23.61279
$t_5 = t_6 = t_9$	207.81647	207.33018	206.98932
t_7	72.84279	72.90264	72.94482
t_8	44.39684	44.46422	44.51177
$t_{10}(\alpha = -2)$			
$\delta = -2$	**220.7079**	**222.9929**	**225.5105**
$\delta = -1$	**214.1674**	**214.9891**	**215.9779**
$\delta = 0$	**210.5338**	**210.5921**	**210.8002**
$\delta = 1$	**208.7200**	**208.4113**	**208.2488**
$\delta = 2$	**207.9876**	**207.5341**	**207.2262**
$t_{10}(\alpha = -1)$			
$\delta = -2$	**210.5338**	**210.5921**	**210.8002**
$\delta = -1$	**208.7200**	**208.4113**	**208.2488**
$\delta = 0$	**207.9876**	**207.5341**	**207.2262**
$\delta = 1$	**207.8175**	**207.3313**	**206.9906**
$\delta = 2$	**207.8552**	**207.3771**	**207.0444**
$t_{10}(\alpha = 0)$			
$\delta = -2$	**207.9876**	**207.5341**	**207.2262**
$\delta = -1$	**207.8175**	**207.3313**	**206.9906**
$\delta = 0$	**207.8552**	**207.3771**	**207.0444**
$\delta = 1$	**207.8911**	**207.4202**	**207.0948**
$\delta = 2$	**207.8552**	**207.3771**	**207.0444**
$t_{10}(\alpha = 1)$			
$\delta = -2$	**207.8552**	**207.3771**	**207.0444**
$\delta = -1$	**207.8911**	**207.4202**	**207.0948**
$\delta = 0$	**207.8552**	**207.3771**	**207.0444**
$\delta = 1$	**207.8175**	**207.3313**	**206.9906**
$\delta = 2$	**207.9879**	**207.5346**	**207.2267**
$t_{10}(\alpha = 2)$			
$\delta = -2$	**207.8552**	**207.3771**	**207.0444**
$\delta = -1$	**207.8175**	**207.3313**	**206.9906**
$\delta = 0$	**207.9879**	**207.5346**	**207.2267**
$\delta = 1$	**208.7221**	**208.4144**	**208.2529**
$\delta = 2$	**210.5418**	**210.6037**	**210.8163**

Table 4: - In this table shows PRE for $b = -0.1$

Estimators	K=2	K=3	K=4
t_1	100.00000	100.00000	100.00000
t_2	100.49521	100.60336	100.71150
t_3	89.55231	89.64193	89.70442
t_4	110.18663	110.19087	110.19383
$t_5 = t_6 = t_9$	110.33942	110.33081	110.32509
t_7	97.16601	97.19237	97.21073
t_8	108.15381	108.12321	108.10193
$t_{10}(\alpha = -2)$			
$\delta = -2$	**110.3653**	**110.3645**	**110.3666**
$\delta = -1$	**110.4344**	**110.4495**	**110.4676**
$\delta = 0$	**110.5266**	**110.4495**	**110.5999**
$\delta = 1$	**110.6249**	**110.6809**	**110.7398**
$\delta = 2$	**110.7156**	**110.7905**	**110.8683**
$t_{10}(\alpha = -1)$			
$\delta = -2$	**110.5266**	**110.5617**	**110.5999**
$\delta = -1$	**110.6249**	**110.6809**	**110.7398**
$\delta = 0$	**110.7156**	**110.7905**	**110.8683**
$\delta = 1$	**110.7880**	**110.8780**	**110.9708**
$\delta = 2$	**110.8346**	**110.9342**	**111.0366**
$t_{10}(\alpha = 0)$			
$\delta = -2$	**110.7156**	**110.7905**	**110.8683**
$\delta = -1$	**110.7880**	**110.8780**	**110.9708**
$\delta = 0$	**110.8346**	**110.9342**	**111.0366**
$\delta = 1$	**110.8508**	**110.9537**	**111.0594**
$\delta = 2$	**110.8349**	**110.9346**	**111.0372**
$t_{10}(\alpha = 1)$			
$\delta = -2$	**110.8346**	**110.9342**	**111.0366**
$\delta = -1$	**110.8508**	**110.9537**	**111.0594**
$\delta = 0$	**110.8349**	**110.9346**	**111.0372**
$\delta = 1$	**110.7886**	**110.8787**	**110.9718**
$\delta = 2$	**110.7162**	**110.7915**	**110.8696**
$t_{10}(\alpha = 2)$			
$\delta = -2$	**110.8349**	**110.9346**	**111.0372**
$\delta = -1$	**110.7886**	**110.8787**	**110.9718**
$\delta = 0$	**110.7162**	**110.7915**	**110.8696**
$\delta = 1$	**110.6255**	**110.6818**	**110.7411**
$\delta = 2$	**110.5271**	**110.5625**	**110.6010**

Table 5: - This table shows PRE for $b = 0.1$

Estimators	K=2	K=3	K=4
t_1	100.00000	100.00000	100.00000
t_2	100.05672	100.06897	100.08121
t_3	49.54164	49.72916	49.86099
t_4	36.77610	36.92607	37.03156
$t_5 = t_6 = t_9$	113.70087	113.66476	113.63967
t_7	79.70503	79.82610	79.91089
t_8	62.30713	62.44795	62.54669
$t_{10}(\alpha = -2)$			
$\delta = -2$	**115.7685**	**116.1479**	**116.5438**
$\delta = -1$	**114.6545**	**114.8056**	**114.9692**
$\delta = 0$	**114.0510**	**114.0819**	**114.1241**
$\delta = 1$	**113.7790**	**113.7571**	**113.7462**
$\delta = 2$	**113.7023**	**113.6663**	**113.6413**
$t_{10}(\alpha = -1)$			
$\delta = -2$	**114.0510**	**114.0819**	**114.1241**
$\delta = -1$	**113.7790**	**113.7571**	**113.7462**
$\delta = 0$	**113.7023**	**113.6663**	**113.6413**
$\delta = 1$	**113.7190**	**113.6871**	**113.6662**
$\delta = 2$	**113.7576**	**113.7337**	**113.7209**
$t_{10}(\alpha = 0)$			
$\delta = -2$	**113.7023**	**113.6663**	**113.6413**
$\delta = -1$	**113.7190**	**113.6871**	**113.6662**
$\delta = 0$	**113.7576**	**113.7337**	**113.7209**
$\delta = 1$	**113.7752**	**113.7550**	**13.7458**
$\delta = 2$	**113.7576**	**113.7338**	**113.7209**
$t_{10}(\alpha = 1)$			
$\delta = -2$	**113.7576**	**113.7337**	**113.7209**
$\delta = -1$	**113.7752**	**113.7550**	**113.7458**
$\delta = 0$	**113.7576**	**113.7338**	**113.7209**
$\delta = 1$	**113.7190**	**113.6871**	**113.6662**
$\delta = 2$	**113.7023**	**113.6663**	**113.6413**
$t_{10}(\alpha = 2)$			
$\delta = -2$	**113.7576**	**113.7338**	**113.7209**
$\delta = -1$	**113.7190**	**113.6871**	**113.6662**
$\delta = 0$	**113.7023**	**113.6663**	**113.6413**
$\delta = 1$	**113.7792**	**113.7573**	**113.7466**
$\delta = 2$	**114.0520**	**114.0834**	**114.1261**

6. Results and interpretation

The numerical study evaluates and compares the performance of the nine existing estimators $t_i(\mathrm{i} = 1, \cdots, 9)$ with the proposed class of estimator t_{10} across four artificially generated normal bi-variate populations, according to $b = -0.3, 0.3, -0.1,\ and\ 0.1$. The different amount of subsampling from non-respondent groups are introduced by putting $k = 2, 3, 4$. The percent relative efficiency (PRE) of all the estimators are calculated with respect to t_1. The following interpretations are made from the results reported in Table2 to Table 5.

In Table 2, $(b = -0.3)$, the baseline estimator t_1 has $PRE = 100$ for all K, while the regression-type estimators t_5, t_6, and t_9 achieve PRE around $198 - 199$, indicating roughly a doubling of efficiency over t_1. The ratio estimator t_3, product estimator t_4, and exponential estimators $t_7$and t_8 remain substantially less efficient, often below 100 PRE. In contrast, members of the proposed class t_{10} attain the highest PRE in this table, with several (α, δ) combinations (particularly , $\delta = -2, -1, 0, 1, 2$) yielding PRE values near or above 199.6 for all K, slightly but consistently exceeding the best existing estimators.

Table 3, $(b = 0.3)$ shows that t_1 remains the reference $(PRE = 100)$, and the regression-type estimators t_5,t_6 and t_9 again dominate the classical ratio, product and exponential estimators, with PRE around 207.8. Here, the product estimator t_4 and exponential estimators t_7 and t_8 showing lesser efficiency. The proposed estimators t_{10} provide modest but systematic improvements, with the best (α, δ) choices attaining improved PRE for all values of K typically lying in the band $208 - 225$, thereby maintaining an efficiency advantage over t_5, t_6, and t_9.

In Table 4, $(b = -0.1)$, the regression-type estimators t_5, t_6, and t_9 achieve PRE around 110.4, substantially above the baseline t_1 and clearly higher than the ratio t_3, product t_4, and exponential estimators t_7 and t_8 . Within this table, the proposed class t_{10} produces PRE values that are generally close to, and at some parameter combinations slightly above, those of t_5, t_6, and t_9, depending on α, δ, and K. Thus, even when the regression relationship is weakly negative, the best members of t_{10} match or marginally exceed the most efficient existing estimators.

Table 5, $(b = 0.1)$ presents a similar pattern: t_1 is fixed at $PRE = 100$, while the regression-type estimators t_5, t_6, and t_9 yield PRE around 113.6 across all K, outperforming the ratio, product, and exponential competitors. The proposed estimators t_{10} again reach the highest PRE values in the table for suitable (α, δ), with efficiencies up to approximately 116 for $K = 2,3,4$, indicating a consistent—though moderate—gain relative to the regression-type estimators when the slope is small and positive.

Overall, across all four tables and for every combination of b and K, the PRE values show that classical ratio and product estimators t_3 and t_4 , as well as exponential estimators t_7 and t_8, are uniformly less efficient than the regression-type estimators t_5, t_6, and t_9 . The proposed class t_{10} always includes at least one configuration of α and δ whose PRE exceeds that of all existing estimators.

7. Conclusion

In this article we have considered nine population mean estimators. The approximate expressions for their Bise and MSE's in presence of measurement error and non-response error are derived. The numerical investigation is made to calculate the MSE for all the estimators for an artificially generated bivariate population by reassigning the value of b four different populations have been considered. The result of numerical investigation is tabulated for different value of α and δ. The numerical investigation shows that our proposed estimator is better than other existing estimators in terms of efficiency.

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